Sampling Theory and Analog-to-Digital Conversion

Introduction/Definitions
Analog-to-digital conversion
Sampling Rate
Frequency Analysis

Basic Measurement Issues
Reliability – the extent to which a measurement procedure yields the same results on repeated trials
Validity – the extent to which a measurement procedure measures what it is supposed to measure
Sensitivity – responsiveness of an instrument to a change in the incoming signal
Precision – the degree of exactness with which a measurement is made, similar meaning as accuracy
Resolution – The fineness of detail that can be distinguished in a measurement

Definitions
Signal – a fluctuating electric quantity, such as a voltage or current, whose variations represent coded information
Analog signal – represents data by a continuously variable quantity
Digital signal – represents data in a discrete, numerical form

Digitization – the process of converting an analog signal into digital form
Analog-to-digital converter (ADC) – a device that digitizes an analog signal
Sampling – an analog signal is said to be sampled to produce the digital signal
Quantization – to subdivide into small but measurable increments: digital signals are limited by the number of binary digits ("bits") that can be used to represent them

Analog-to-digital conversion process

Parameters of the ADC process:
Input range – establishes max voltage range to be detected
– bipolar: ±1V, ±5V, ±10V
– unipolar: 0-1V, 0-5V, 0-10V
– usually selectable in hardware or software
Input configuration
– single-ended: one input relative to ground
– differential: difference between 2 inputs is sampled
System gain – degree to which signal is amplified before sampling
Analog-to-digital conversion

Parameters of the ADC process:

Amplitude resolution – how precisely amplitude of signal can be quantified
- determined by "bit value" of ADC
- common: 12-bit ($2^{12} = 4096$ discreet levels)
- 16-bit ($2^{16} = 65,536$ discreet levels)

Temporal resolution – how frequently the ADC takes samples of the analog signal
- determined by "sampling rate"
- values from 30 Hz to 2000 Hz are common

Together, amplitude and temporal resolution greatly affect the quality of the sampled signal.

ADC – Amplitude Resolution

A 4-bit A/D set up ($2^4 = 16$ discreet levels)

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Binary Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.375 V</td>
<td>1101</td>
</tr>
<tr>
<td>3.750 V</td>
<td>1100</td>
</tr>
<tr>
<td>3.125 V</td>
<td>1111</td>
</tr>
<tr>
<td>2.500 V</td>
<td>1101</td>
</tr>
<tr>
<td>1.875 V</td>
<td>1100</td>
</tr>
<tr>
<td>0 V</td>
<td>1011</td>
</tr>
<tr>
<td>1.250 V</td>
<td>1010</td>
</tr>
<tr>
<td>0.625 V</td>
<td>1010</td>
</tr>
<tr>
<td>0 V</td>
<td>1001</td>
</tr>
<tr>
<td>-1.250 V</td>
<td>1011</td>
</tr>
<tr>
<td>-1.875 V</td>
<td>1010</td>
</tr>
<tr>
<td>-2.500 V</td>
<td>1010</td>
</tr>
<tr>
<td>-3.125 V</td>
<td>1011</td>
</tr>
<tr>
<td>-3.750 V</td>
<td>1010</td>
</tr>
</tbody>
</table>

ADC – Temporal Resolution

Sampling Rate: how often is the signal sampled?

Sampling period: time between adjacent samples
- $t = t_2 - t_1$

Sampling rate: frequency with which samples are taken
- $f = 1 / t$

What is an adequate sampling rate?

Governed by Shannon’s sampling theorem:

“the signal must be sampled at a frequency at least twice as high as the highest frequency present in the signal itself”

This minimally acceptable sampling frequency is known as the Nyquist frequency

ADC – Amplitude Resolution

How precisely does a 12-bit A/D board detect amplitude changes in a signal?

12 bit ADC = $2^{12} = 4096$ discreet levels, distributed over the whole input range

For a ±10 V input, you have a range of 20 V

$(20 \text{ V}) / (4096 \text{ ADC units}) = 0.0049 \text{ V per ADC unit}$

Is 4.9mV per ADC unit good or bad resolution?

Force Plate: if +10 V = 2500 N (for a given FP gain setting), then 1 ADC unit = 1.22 N

In other words, you will be unable to detect changes in force that are less than 1.22 N, this is probably OK

ADC – Temporal Resolution

What exactly does signal frequency content mean?
Fourier (Harmonic) Analysis

- The frequency content of a signal can be determined by performing a Fourier analysis.
- Fourier discovered that any signal, no matter how complex, can be represented by an infinite sum of weighted sine and cosine waves.
- In practice, only a finite number of harmonics are needed to adequately represent a signal.
- A related technique, the Fourier transform, allows the frequency components of a signal to be revealed.

The sine function

\[ y(t) = A + B \sin(f_1 t) + C \cos(f_1 t) + B \sin(f_2 t) + C \cos(f_2 t) + \ldots \]

Approximation of the data improves as the number of harmonics used increases.
However, 94% of the total signal power was contained in just the 1st harmonic.

The Fourier Transform

- Converts signals from the time domain to the frequency domain.
- The Fourier transform separates a signal into sinusoids of different frequency which sum to the original waveform; it distinguishes the different frequency sinusoids and their respective amplitudes.
  - Discrete Fourier Transform (DFT)
  - Fast Fourier Transform (FFT)
- Typically, the square of the FFT is used instead, which is called the Power Density Spectrum.
**Fourier Transform**

- **Original Signal:**
  - Time (s)
  - Frequency (Hz)

- **20 Hz Noise Added:**
  - Time (s)
  - Frequency (Hz)

- **FFT:**
  - Frequency (Hz)

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**Frequency Content**

Frequency content of some common signals encountered in biomechanics:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Max Freq of Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standing posture</td>
<td>3 Hz</td>
</tr>
<tr>
<td>Walking</td>
<td>6 Hz</td>
</tr>
<tr>
<td>Running</td>
<td>10-15 Hz</td>
</tr>
<tr>
<td>Heelstrike transient</td>
<td>100-300 Hz</td>
</tr>
<tr>
<td>EMG (surface)</td>
<td>400-500 Hz</td>
</tr>
<tr>
<td>EMG (indwelling)</td>
<td>800-1000 Hz</td>
</tr>
</tbody>
</table>

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**ADC – Temporal Resolution**

Now, back to sampling rate:

- What happens if you sample at a rate lower than the Nyquist frequency?
- An error, known as aliasing, will occur, corrupting your data (aliased signals show up at a freq of \( f_{samp} - f_{sig} \), so 40Hz signal sampled at 50Hz will show up as a 10Hz signal)
- When aliasing occurs, frequencies greater than \( \frac{1}{2} \) the sampling rate “fold back” into the lower frequency components, distorting the signal
- High frequency information is not simply lost, it actually reappears as false low frequency content

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**Sampling**

- “One common form of aliasing, which most people have observed, is the ‘wagon wheels’ effect in films. Since the rate of the film is usually much lower than the frequency at which the spokes of a wheel pass any one point, aliasing takes place and the wheels appear to turn either much more slowly than they really are, or even seem to go backwards sometimes.” (D.W. Grieve et. al., *Techniques for the Analysis of Human Movement*. Princeton Book Company, 1975, p. 41.)

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**Sampling Diagram**

Figure 2.15 Sampling of two signals, one at a proper rate, the other at too low a rate. Signal 2 is sampled at a rate less than twice its frequency, which as its sampled amplitudes are the same as for Signal 1. This represents a violation of the sampling theorem and results in an error called aliasing.
Sampling

Sampling Frequency

Another example of aliasing:
1 Hz signal appears to be 0.33 Hz

Sampling Rate

Take home message(s):
• Sampling at the Nyquist frequency ensures that frequency content of signal is preserved
• In practice, sampling at 4-5 times the highest frequency present in the signal ensures that signal amplitude characteristics are preserved as well
• When possible, an “anti-aliasing” filter should be used to eliminate noise that exists above ½ the sampling rate

Sampling Rate

Typical sampling rates used to collect various signals in biomechanics:

<table>
<thead>
<tr>
<th>Signal</th>
<th>Typical SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motion</td>
<td>30-240 Hz</td>
</tr>
<tr>
<td>Force</td>
<td>100-1000 Hz</td>
</tr>
<tr>
<td>EMG (surface)</td>
<td>1000-2000 Hz</td>
</tr>
<tr>
<td>EMG (indwelling)</td>
<td>2000-5000 Hz</td>
</tr>
</tbody>
</table>

Sampling

• How do we determine the length of sampling period?
• In repetitive motion the time needs to be more than the time for 1 cycle so if gait is 120 steps (60 strides) per minute then the rate is 1Hz and you need to sample longer than 1s to obtain the full data set.
• You may increase time to collect multiple repetitions.
Sampling

- In non repetitive activities such as lifting, then you need to go from start to finish.
- In cases where frequency of movement is not obvious, pilot data needed to enable appropriate decision. Winter reported that in quiet standing, significant data found at .002Hz which would require an 8 minute trial.

Measurement Errors

- No measurement process is perfect, there will be some degree of error in the data.
- Measurement errors fall into two categories:
  - Random Errors
  - Systematic Errors
- Both can be minimized, but neither can be completely eliminated.
- In particular, random errors may be reduced through data processing techniques, systematic errors generally can not.

Random Error - Example

3 people measure the distance between two lines that are 1.5 cm apart.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5 cm</td>
</tr>
<tr>
<td>2</td>
<td>1.4 cm</td>
</tr>
<tr>
<td>3</td>
<td>1.6 cm</td>
</tr>
</tbody>
</table>

They don’t all get the same value, but this “random fluctuation” can be largely eliminated by averaging the three values

\[(1.5 + 1.4 + 1.6) / 3 = 1.5 \text{ cm}\]

Systematic Error - Example

1 person measures the distance between two lines that are 1.5 cm apart, but places the ruler down incorrectly.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2 cm</td>
</tr>
<tr>
<td>2</td>
<td>1.2 cm</td>
</tr>
<tr>
<td>3</td>
<td>1.2 cm</td>
</tr>
</tbody>
</table>

This individual was very consistent in his measurements, but he has introduced a systematic error into the data.

No amount of processing can remove this error.

Measurement Errors in Biomechanics

Systematic Errors: cover frequency spectrum
- Poor placement of anatomical markers
- Movement of markers/skin relative to bones
- Calibration nonlinearities
- Digitization errors (some)

Random Errors: typically high frequency
- Digitization errors (some)
- Marker jiggling/vibration
- Electrical interference

Minimizing Measurement Errors

- Use of consistent, proper techniques for data collection
- Use of 3-D techniques for motion capture
- Proper training of personnel
- Use of high quality equipment
Up Next…

Topic
– Kinematics

Readings